Combinatorial Design Homework (III)

12,6−12,27

1. Find the maximum packing of the complete graph with triangles.
2. Prove that $K_3|\lambda K_v$ if and only if (i) $\lambda(v-1)$ is even and (ii) $\binom{\lambda v}{2}$ is a multiple of three.
3. Prove that for each $m \equiv 1 \ or \ 3 \ (mod \ 6), m \geq 3$, $K_{m(n)}$ can be decomposed into triangles.
4. Prove that $K_{m(n)}$ can be decomposed into triangles if and only if $m \geq 3$ and $K_{m(n)}$ is $3 - sufficient$.
5. Find a PBD with $v = 22$, $\lambda = 1$ and $K = \{4, 7\}$.
6. Prove that if $n \in GD(S, R\ ; 1)$, $mR + 1 \subseteq B(k, \lambda)$ and $mS \subseteq GD(m, -; k, \lambda)$, then $mn + 1 \in B(k, \lambda)$.
7. Let $r \in GD(K, M\ ; \lambda)$, $M \subseteq T(s, \lambda)$ and $K \subseteq RT(s, 1)$. Show that $r \in T(s, \lambda)$.
8. Use the conclusion roved in 5. to show that there exists a pair of orthogonal latin squares of order 22.
9. Prove that there are three orthogonal latin squares of order 101.
10. Prove that a $(v, 4, 1)-$design exists if and only if $v \equiv 1 \ or \ 4 \ (mod \ 12)$.