

Combinatorial Design Homework (II)

11,8-?,?

1. Prove that $\lambda(v-1) \geq k(k-1)$ in a $2-(v, k, \lambda)$ design.
2. Let (\mathbb{X}, \mathbb{B}) be a pairwise balanced design. Prove that $|\mathbb{B}| \geq |\mathbb{X}|$.
3. Give five examples such that both $\frac{\lambda v(v-1)}{k(k-1)}$ and $\frac{\lambda(v-1)}{k-1}$ are positive integers but no $2-(v, k, \lambda)$ designs exist.
4. Use recursive construction to prove that a $2-(v, 3, 1)$ design exists if and only if $v \equiv 1$ or $3 \pmod{6}$.
5. Use difference method to prove the statement in 4.
6. Use direct constructions to prove the statement in 4.
7. Let (\mathbb{X}, \mathbb{P}) be a packing of $K_{|\mathbb{X}|}$ with triangles. Find $\max |\mathbb{P}|$.
- (*) 8. Let (\mathbb{X}, \mathbb{B}) be a steiner triple system of order v . Define a *STS* graph G on (\mathbb{X}, \mathbb{B}) by letting $V(G) = \mathbb{B}$ and two vertices in G are adjacent if and only if their intersection is not empty. Find $\chi(G)$.
9. Find two *STS*(19) which has exactly three triples in common.
10. Find a resolvable *STS*(21).