

Algebra (I), Test 3.

11,30

1. Find a group of order 12 which contains no subgroups of order 6. (20 points)
2. Find all abelian groups, up to isomorphism, of order 240. (20 points)
3. Give four examples of homomorphisms. (20 points)
4. Let $\varphi : G \rightarrow G'$ be a homomorphism. Prove the following statements. (40 points)
 - (a) $\varphi[G] \leq G'$.
 - (b) For each subgroup K' of G' , $\varphi^{-1}[K'] \leq G$.
 - (c) $\text{Ker}(\varphi)$ is a normal subgroup of G .
 - (d) $G/\text{Ker}(\varphi)$ is a factor group.
 - (e) φ is a one-to-one map if and only if $\text{Ker}(\varphi) = e$.
 - (f) $G/\text{Ker}(\varphi)$ is isomorphic to $\varphi[G]$.
5. Prove that if m divides the order of a finite abelian group G , then G has a subgroup of order m . (20 points)