Algebra (I), Test 4.

12,28

20 points each.

1. Give the definition of "group action" and then give an example to explain the notion.
2. Give two examples of ring homomorphism. (Verify your answers.)
3. Give a ring R such that $\forall\ a \in R, a^2 = a$.
4. Show that the characteristic of an integral domain must be either 0 or a prime p.
5. Prove that if a is an integer relatively prime to n, then $a^{\varphi(n)} \equiv 1\ (mod\ n)$ where $\varphi(n)$ is the number of integers in $(0, n)$ which are relatively prime to n.
6. Let p be a prime. Prove that $(p-1)! \equiv -1\ mod\ p$.
7. Give a division ring which is not a field. (Verify your answer.)