Algebra (I), Test 2.

11,9

Choose 6 questions of below, each 20 points

1. Find a group of order 10 which is not abelian. (Verify your answer.)
2. Prove that every permutation can be expressed as a product of transpositions and the number of transpositions can not be both even and odd.
3. State and prove Theorem of Lagrange.
4. Prove or disprove that $Ha = aH$ for each $a \in G$ and $H \leq G$.
5. Prove that $n = \sum_{d|n} \phi(d)$.
6. Show that for every subgroup H of S_n for $n \geq 2$, either all the permutations in H are even or exactly half of then are even.
7. Show that $\mathbb{Z}_3 \times \mathbb{Z}_3$ and $\mathbb{Z}_9$ are not isomorphic.
8. Prove that every group of prime order is abelian.