Combinatorial Design Homework (IV)

12,28−1,?

1. Construct an $SQS(10)$ and an $SQS(14)$. Is the $SQS(14)$ you construct 2-chromatic?
2. Construct an $SQS(4v)$ from an $SQS(v)$ by using the notion of a latin cube. (A latin cube of order n is a collection of n latin squares $L_1, L_2, \cdots, L_n$ of order n such that for each (i, j), $\{L_1(i, j), L_2(i, j), \cdots, L_n(i, j)\}$ is a set of n distinct elements $\mathbb{Z}_n$).
3. Find a necessary condition of two $SQS(v)$ which are isomorphic.
4. Prove that K_{2m} has a 1-factorization by using the maximum packing of K_{2m+1} with triangles.
5. Prove that $C_5|K_n$ if and only if $n \equiv 1$ or 5 $(mod\ 10)$ and $n \neq 6$.

(∗) 6. Prove that $C_{2k}|K_{m,n}$ if and only if $min\{m, n\} \geq k$, both m and n are even and $2k|mn$.

7. Let G be a 2-regular subgraph of K_n. Then $C_4|K_n - G$ if and only if $K_n - G$ is 4-sufficient.
8. Let n be an odd integer such that $3a + 4b = \binom{n}{2}$. Prove that K_n can be decomposed into a C_3's and b C_4's.
9. Find the maximum packing of K_n with 6-cycles.
10. Find the maximum packing of K_n with paths of length 5.