Graph Theory Exercise (II) Due Nov. 24

(5 points each for the following exercises)

1. Find a proof of Menger's Theorem which is different from the proof given in Lecture 6.

2. Use maximum flow - minimum cut theorem to prove Hall's Theorem.

3. A non-negative nxn matrix $A = (a_{ij})$ is said to be doubly stochastic if both row sum and column sum are equal to 1. Prove that there exist positive real numbers d_k, k = 1, 2, ..., m with their sum is 1 and permutation matrices P_k's such that $A = d_1P_1 + d_2P_2 + \ldots + d_mP_m$.

4. Show that Petersen graph does not contain a Hamilton cycle.

5. Let Z be an arbitrarily given 2-factor with n vertices. Prove that the circulant graph G(n; {1,2}) can be decomposed into a 2-factor Z and a Hamilton cycle.

6. Find a good lower bound and a good upper bound for R(5, 5).

7. Let a graph G be 2-cell embedded on an orientable surface with n handles. Prove that if G has p vertices, q edges and f faces, then $p - q + f = 2 - 2n$. (Euler-Poincare theorem)

8. (Bonus) Determine $Cr(K_8)$ and $Cr(K_{8,8})$.