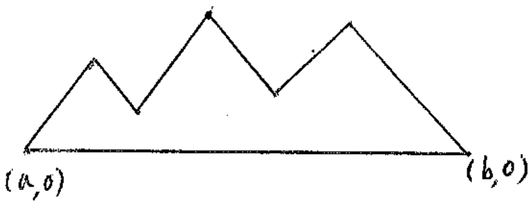
Graph Theory Exercise (I) Due Oct. 20, 2022

(*) The following exercises are 5 points each.

1. Hikers A and B begin to travel at (a, 0) and (b, 0) respectively on a mountain range. Prove that if they travel in such a way that at all times their heights are the same, then they can meet somewhere between their starting points. Define a graph to model the movements and claim the statement.



2. Use the adjacency matrix of a graph G, to find the number of 4-cycles (2 points) and the number of 5-cycles (3 points) in G.
3. Find the size of an extremal graph of order 101 which does not contain a complete graph of order 11 as a subgraph. That is, find ex(101; K_{11}).
4. Prove that every connected graph G contains a path or cycle of length at least min. $\{2\cdot\delta(G), |G|\}$. Moreover, if $\delta(G) \geq |G|/2$, then G contains a Hamilton cycle.
5. For positive integers a, b and $2a \geq b \geq a$, construct a 2-connected graph G such that rad(G) = a and diam(G) = b.
6. Prove that for each bipartite graph G of size 16, there exists an induced subgraph of G whose size is 8.
7. Prove or disprove that a graph G can be decomposed into k bipartite subgraphs where $k \leq \log_2 |E(G)|$.
8. (Bonus) Find ex(n; C_4) for as many n as possible.