Combinatorial Design Homework (I)

9,20−10,10

1. Let $L(n)$ denote the number of distinct latin squares of order n. Prove that $L(n) \geq \prod_{k=1}^{n} k!$ and find $L(5)$.

2. A quasigroup $< Q, * >$ is totally symmetric if for each pair of element a and b, $a * (a * b) = b$ and $(b * a) * a = b$. Prove that $a * b = b * a$ and a totally symmetric quasigroup of order n exists for each $n \in \mathbb{Z}^+$.

3. Let n be a prime power. Prove that there are $n + 1$ $n \times n$ arrays based on $\mathbb{Z}_n$ which are mutually orthogonal. Also, find three mutually orthogonal latin squares of order 4. (show your work.)

4. Find four latin squares L_1, L_2, L_3 and L_4 of order 100 such that $L_1 \perp L_2, L_2 \perp L_3, L_3 \perp L_4, L_1 \not\perp L_3, L_1 \not\perp L_4, L_2 \not\perp L_4$.

5. $\forall$ $n \geq 5$, prove that there exists a pair of latin squares of order n, L and M, such that $|L \cap M| = 1$.

6. Let $\mathcal{C}$ be a critical set of a latin square of order n. Prove that $\mathcal{C}$ contains at least $n + 2$ elements for each $n \geq 5$.

7. Let $\mathcal{P}$ be a maximal partial latin square of order 10. Prove that $|\mathcal{P}|$ can be $98, 95, 92, 89, \cdots, 68$.

8. Let L be an arbitrary latin square of order n. Prove that L contains a partial transversal which has at least $\frac{3}{4}n$ entries.

9. Construct a 2^4−Room square.

10. Let $M(n)$ denote the number of mutually orthogonal latin square of order n. Prove that $M(n) = n - 1$ if $M(n) \geq n - 2$.