

Introduction to Combinatorics Term Paper (III) Due June 22nd

1. Given a set S of $2n - 1$ distinct integers. Prove that there exists an n -subset T of S such that the sum of integers in T is a multiple of n .
2. Give a couple of examples satisfying $|A + A + A \cdot A| > |A|^2 + 2|A|$ where A is a set of integers.
3. Let A and B be additive sets in a finite additive group G such that $|A| + |B| > |G|$. Prove that $A + B = A - B = G$. Moreover, give an example to show that the condition $|A| + |B| > |G|$ cannot be improved.
4. In a transportation network, prove that the maximum value of flows is equal to the minimum value of capacity of cuts. (Ford and Fulkerson's theorem)
5. Prove P. Hall's marriage theorem by using Ford and Fulkerson's theorem.
6. Let $cr(G)$ denote the minimum number of crossings for a proper drawing of G on a plane. Determine $cr(K_7)$ and $cr(K_8)$.
7. Let G be a graph obtained by using random graph model $M(n; p)$ where G is of order n and the existence of each edge in G is of probability $p = 1/100$. Prove that almost all graphs G are of diameter 2 and 100-connected.
8. What is "Lovasz Local Lemma"? Give an example to apply the lemma.