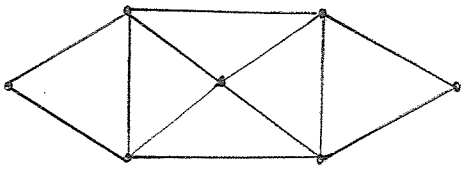
Fundamental Graph Theory Final 2019, 1, 10

1. Give the definitions of the following terms for a given graph G. (5 points each)

   (a) Dominating set
   (b) Class 1 graph
   (c) Prime sum labeling
   (d) Vertex cover
   (e) Crossing number
   (f) Anti-magic labeling
   (g) 1-factor

2. Explain why Petersen graph is not a planar graph? (8 points)
3. What is the chromatic index of the complete graph of order 111? (8 points)
4. What is the Hall's condition of a bipartite graph? Give an example to show how to use the condition. (8 points)
5. Fine a prime difference labeling of the cycle of order 20 by using the integers in {1, 2, ..., 20}. (8 points)
6. Find a vertex distinguished labeling (coloring version) of the following graph. (8 points)



(*) Choose three of the following statements to prove.

7. Prove that there are exactly five regular polyhedra. (15 points)
8. Let $\alpha(G)$ and $\sigma(G)$ denote the independence number and vertex cover number respectively. Prove that $\alpha(G) + \sigma(G) = |G|$. (15 points)
9. Prove that the chromatic index of Petersen graph is 4 and use this result to show that Petersen graph contains no Hamilton cycles.
10. Prove that if $m \geq n$, then the chromatic index of $K_{m,n}$ is equal to m.