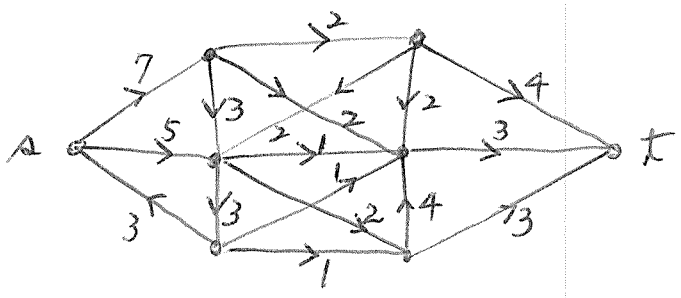
Fundamental Graph Theory Mid-Term 2018, 11, 7

1. (25 points) Give the definitions of the following graph terms.
   (a) Eccentricity of a vertex
   (b) Hamilton cycle
   (c) Degree sequence
   (d) Minimum spanning tree
   (e) K-connected graph

2. (10 points) Find a graph T with diameter 11 and radius 7.

3. (10 points) Find all non-isomorphic graphs of order 6 and size 6.

4. (10 points) Find the maximum flow of the following network. (Explain your answer.)



5. (15 points) Prove that if a graph of order n which does not contain a 3-cycle, then the graph has at most $[n^2/4]$ edges.

6. (15 points) Prove that there exists a directed Hamilton path in a tournament.

7. (15 points) Prove that a connected graph has an Eulerian circuit if and only if each vertex of the graph is of even degree.

8. (15 points, bonus) Let A be the adjacency matrix of a graph G and $V(G) = \{v_1, v_2, \ldots, v_n\}$. Prove that the number of k-walks from v_i to v_j is equal to the (i, j)-entry of A^k.