Basic Terms (Definitions)

The best way to understand the terms is to give an example for each term.

1. Graph (General)

   A **graph** $G = (V, E)$ consists of two sets V and E.

   - The elements of V are called **vertices** (or **nodes**).
   - The elements of E are called **edges**. (E is a collection of 1-subsets or 2-subsets.)
   - Each edge has a set of one or two vertices associated to it, which are called its **endpoints**. An edge is said to **join** its endpoints. (Two endpoints or end vertices of an edge are **adjacent**.)

2. Edge-types

   - A **loop** is an edge that joins a single endpoint to itself.
   - A **multiple edge** is a collection of two or more edges having identical endpoints.

3. Finite and Infinite graph

   A **finite graph** is a graph with a finite number of vertices and edges. A graph which is not finite is called **infinite**.

4. Hypergraph

   A **hypergraph** is a generalization of a graph, where an edge can contain any number of vertices.

5. Multigraph and Pseudograph

   - A **multigraph** is a graph which is permitted to have multiple edges, that is, edges that have the same endpoints.
   - A **pseudograph** is a non-simple graph in which both graph loops and multiple edges are permitted.

6. Simple graph

   A **simple graph** is a graph that has no self-loops or multi-edges.

7. Subgraph and Supergraph

   A graph H is called a **subgraph** of graph $G = (V, E)$ if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. And G is a **supergraph** of H.

8. Edge-deletion

   $G = (V, E)$ is a graph, $e \in E$, then $G - e$ is defined by $G - e = (V, E \setminus \{e\})$.

9. Vertex deletion

   A graph $G = (V, E)$, $v \in V$, then $G - v$ is defined by $G - v = (V \setminus \{v\}, E')$, where $E' = \{e | e \in E$ and v is not the endpoint of $e\}$.

10. Induced subgraph

    Let $G = (V, E)$ be any graph, and let $S \subset V$ be any subset of vertices of G. Then the **induced subgraph** $< S >_G$ is the graph whose vertex set is S and whose edge set consists of all of the edges in E that have both endpoints in S..

11. Edge induced subgraph

    Each subset $E' \subseteq E$ defines a unique subgraph $H = (V', E')$ of graph $G = (V, E)$, where V' consists of only those vertices which are the endpoints of the edges in E'. The **edge-induced subgraph** H is denoted $< E' >_G$.

12. Neighborhood

    The **neighborhood** of a vertex v of a graph is the set of all vertices adjacent to v. It is denoted by $N(v)$.

13. Closed neighborhood

    The **closed neighborhood** is denoted $N[v] = N(v) \cup \{v\}$, where $N(v)$ is the neighborhood of a vertex v in a graph G.

14. Degree

    The **degree** (or **valency**) of a vertex v in a graph G, denoted $deg_G(v)$, is the number of **proper edges** (two vertices) incident to v plus twice the number of self-loops. (For simple graphs, of course, the degree is simply the number of neighbors.) The **maximum degree** of graph G is $\triangle(G) = max\{deg(v) \mid v \in V(G)\}$ and the **minimum degree** of G is $\delta(G) = min\{deg(v)|v \in V(G)\}$.

15. Volume

    A graph $G = (V, E)$, the **volume** of G is $\sum_{v \in V(G)} deg(v)$.

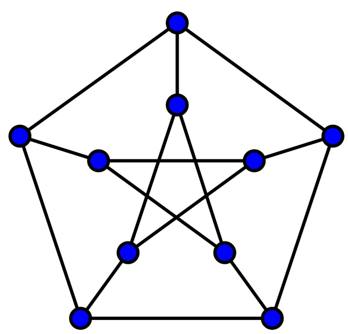
16. Regular graph

    A graph is **regular** if every vertex is of the same degree.

    It is **r-regular** if every vertex is of degree r. And r is called the **valency** of G.

17. Petersen Graph

    Petersen graph is an undirected 3-regular graph with 10 vertices and girth 5 as this figure.



18. Order and Size

    In a graph $G = (V, E)$, the **order** of G is the number of vertices in G, denoted $|G|$. And the **size** of G is the number of edges in G, denoted $||G||$.

19. Complete graph

    A **complete graph** is a graph in which every pair of vertices is joined by an edge.

20. Isolated vertex

    An **isolated vertex** in a graph is a vertex of degree 0.

21. Union and Intersection

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs.

- $G = (V, E)$ is the **union** of G_1 and G_2, if $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$.
- $G = (V, E)$ is the **intersection** of G_1 and G_2, if $V = V_1 \cap V_2$ and $E = E_1 \cap E_2$.

22. Decomposition

If all the followings are satisfied, then we said $\{G_1, G_2, \cdots, G_t\}$ is a **decomposition** of G.

- For all $i \in 1, 2, \cdots, t$, $V(G_i) \subseteq V(G)$.
- $E(G_1) \cup E(G_2) \cup \cdots \cup E(G_t) = E(G)$.
- For any $i \neq j$, $1 \leq i, j \leq t$, $E(G_i) \cap E(G_j) = \emptyset$.

23. Cartesian Product (Square product)

The **cartesian product** (or square **product**) of two graphs G and H is denoted by $G\Box H$, where $V(G\Box H) = V(G) \times V(H)$ and two vertices (x, y) and (x', y') are adjacent if and only if either $x = x'$ and $\{y, y'\} = yy' \in E(H)$ or $y = y'$ and $\{x, x'\} = xx' \in E(G)$.

24. Join

The **join** (or **suspension**) of two graphs G and H is denoted by $G \vee H$, where

$$V(G \vee H) = V(G) \cup V(H) \text{ and}$$
$$E(G \vee H) = E(G) \cup E(H) \cup \{uv | u \in V(G) \text{ and } v \in V(E)\}.$$

25. Composition

The **composition** $G[H]$ of a graph G with a graph H is the graph with vertex set $V(G) \times V(H)$ such that (u_1, v_1) is adjacent to (u_2, v_2) whenever either u_1 is adjacent to u_2, or v_1 is adjacent to v_2 with $u_1 = u_2$.

26. Complement

The **complement** (or **edge-complement**) $\overline{G} = (V, \overline{E})$ of a simple graph $G = (V, E)$ has the same vertex set V as G and edges are defined : e is in $\overline{E}$ if and only if e is not in E.

27. Line graph (G is simple.)

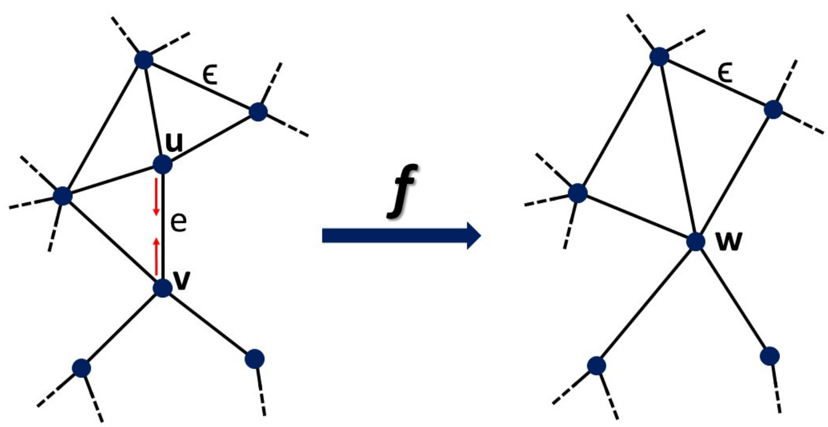
The **line graph** $L(G)$ of a graph G has the edge set of G as its vertex set, i.e. $V(L(G)) = E(G)$; two vertices of $L(G)$ are adjacent if the edges in G to which they correspond have a common vertex, i.e. $E(L(G)) = \{ef | e, f \in E(G) \text{ and } |e \cap f| = 1\}$.

28. Edge Contraction

Let $G = (V, E)$ be a graph containing an edge $e = uv$ with $u \neq v$. Let f be a function which maps every vertex in $V \setminus \{u, v\}$ to itself, and otherwise, maps it to a new vertex w. The contraction of e results in a new graph $G' = (V', E')$, where $V' = (V \setminus \{u, v\}) \cup \{w\}$, $E' = E \setminus \{e\}$.

(a) $f(x) = x$ if $x \in V \setminus \{u, v\}$, and $f(u) = f(v) = w$.

(b) $f(\epsilon) = \epsilon$ if $\epsilon \bigcap \{u, v\} = \emptyset$, $f(zu) = zw$ and $f(zv) = zw$ if zu and zv are edges of G.



29. Contraction graph

H is a **contraction graph** of G if H is obtained by contracting some edges in G.

30. Minor

A **minor** of a graph G is a graph which can be obtained from G by deleting some vertices and edges, and then contracting some further edges.

31. Isomorphism

An **isomorphism between two simple graphs** G and H is a vertex bijection $\phi : V(G) \to V(H)$ such that for $u, v \in V(G)$, the vertex u is adjacent to the vertex v in graph G if and only if $\phi(u)$ is adjacent to $\phi(v)$ in graph H. Implicitly, there is also an edge bijection $E(G) \to E(H)$ such that $uv \mapsto \phi(u)\phi(v)$. We say that G and H are **isomorphic graphs** and we write $G \cong H$.

32. Degree sequence

The **degree sequence** of a graph G is the sequence of its degrees of vertices, arranged from the largest value to the smallest.

33. Automorphism (An isomorphism to itself.)

Given a graph X, a permutation α of $V(X)$ is an **automorphism** of X if

$$\{u, v\} \in E(X) \Leftrightarrow \{\alpha(u), \alpha(v)\} \in E(X), \text{ for all } u, v \in V(X).$$

34. Self-complementary

A **self-complementary** graph is a graph which is isomorphic to its complement.

35. Graphical sequence

We call a sequence of non-negative integers $d_1, \ldots, d_n$ **graphical** if there exists a graph G of order n such that the vertices of which have, in some order, degrees $d_1, \ldots, d_n$.

36. Distance ($d_G(u, v)$)

The **distance** between two vertices u and v in a graph G, $d_G(u, v)$, is the length of the shortest walk between them. If there are no path between two vertices, then the distance of these two vertices is infinite. (A **walk** in a graph G is a sequence of vertices $< v_1, v_2, \cdots, v_m >$ such that $v_iv_{i+1} \in E(G)$, $i = 1, 2, \cdots, m-1$.)

37. Eccentricity

The **eccentricity** of a vertex v in a connected graph is its distance to a vertex farthest from v. Denote $e_G(v) = \max\{d_G(u, v) | u \in V(G)\}$.

38. Diameter and Radius

    The **diameter** of a connected graph G is its maximum eccentricity. Denote

$$D(G) = \max\{e_G(v) | v \in V(G)\}.$$

    The **radius** of a connected graph G is its minimum eccentricity. Denote

$$r(G) = \min\{e_G(v) | v \in V(G)\}.$$

39. Center

    The **center** of a graph is the subgraph induced on its set of the vertices whose eccentricity equals the radius of the graph.

40. Connected graph

    A graph is **connected** if between every pair of vertices there is a walk. If a graph is not connected, then it is **disconnected**.

41. Acyclic, Forest, and Tree

    - A graph is **acyclic** if it has no cycles.
    - A **forest** is an acyclic graph.
    - A **tree** is an acyclic connected graph.

42. Spanning subgraph and Spanning tree

    - A subgraph H of a graph G is a **spanning subgraph** if $V(H) = V(G)$.
    - A **spanning tree** of a graph G is a spanning subgraph of G that is a tree.

43. Component and Trivial component

    - A **component** of a graph G is a connected subgraph H such that no subgraph of G that properly contains H is connected. In other words, a component is a *maximal* connected subgraph.
    - A component (or graph) is called **trivial** if it consists of one vertex.

44. Bridge graph and Bridgeless graph

- A **bridge** of a connected graph is a graph edge whose removal disconnects the graph.
- A **bridgeless graph** is a graph that contains no (graph) bridges.

45. Digraph

A **digraph** (or **directed graph**) is a graph each of whose edges are directed. An arc (x, y) is considered to be directed from x to y; y is called the **head** and x is called the **tail** of the arc; y is said to be a direct **successor** of x and x is said to be a direct **predecessor** of y.

46. Outdegree and Indegree

D is a digraph.

- The **outdegree** of a vertex v in D is the number of arcs directed **from** v.
- The **indegree** of vertex v is the number of arcs directed **to** v.

47. Weakly connected, Strongly connected, Strong component

- A digraph is **weakly connected** if its underlying graph is connected.
- A digraph is **strongly connected** if every two vertices are mutually reachable.
- A **strong component** of a digraph G is a maximal strongly connected subdigraph of G.

48. Orientation, Oriented graph, and Tournament

An **orientation** of an undirected graph assigns a unique direction to each edge. An **oriented graph** is a digraph obtained by choosing an orientation for each edge of an undirected simple graph. A **tournament** is an oriented complete graph.

49. King

A **king** in a tournament T is a vertex x with maximum out-degree.

50. Weighted graph

A **weighted graph** is a graph in which each edge is given a numerical weight. A weighted graph is therefore a special type of labeled graph in which the edge labels are numbers (which are usually taken to be positive).

51. Adjacency matrix and Incidence matrix

- An **adjacency matrix** for a simple graph G whose vertices are explicitly ordered $v_1, v_2, \cdots, v_n$ is the $n \times n$ matrix A_G such that

$$A_G(i,j) = \begin{cases} 1, & \text{if } v_i \text{ and } v_j \text{ are adjacent; and} \\ 0, & \text{otherwise.} \end{cases}$$

- An **incidence matrix** for a simple graph G whose vertices are explicitly ordered $v_1, v_2, \cdots, v_n$ and edges are explicitly ordered $e_1, e_2, \cdots, e_m$ is the $n \times m$ matrix B_G such that

$$B_G(i,j) = \begin{cases} 1, & \text{if } v_i \text{ and } e_j \text{ are incident; and} \\ 0, & \text{otherwise.} \end{cases}$$

52. Cycle matrix

If $|E(G)| = m$, and $C_1, C_2, \ldots, C_q$ are all the circuits in graph G. Then the **cycle matrix** of G is a $q \times m$ matrix $C(G) = (a_{ij})$, and

$$a_{ij} = \begin{cases} 1, & \text{if } e_j \text{ is an edge of } C_i, \text{ and} \\ 0, & \text{if } e_j \text{ otherwise.} \end{cases}$$

53. Walk, Trial and Internal vertex

- A **walk** in a graph G is an alternating sequence of vertices and edges,

$$W = v_0, e_1, v_1, e_1, \ldots, e_n, v_n,$$

such that for $j = 1, \ldots, n$, the vertices v_{j-1} and v_j are the endpoints of the edge e_j.

- A **trail** in a graph is a walk such that no edge occurs more than once.
- A **u-v walk** is defined as a sequence of vertices starting at u and ending at v, where consecutive vertices in the sequence are adjacent vertices in the graph.
- A **u-v trail** is a u-v walk, where no edge is repeated (each edge is used at most once).

- A **u-v path** is a u-v walk, where no vertex is repeated (each vertex is used at most once).
- An **internal vertex** is a vertex of degree at least 2.
- A closed trail is a **circuit**.
- A **path** in a graph is a trail such that no internal vertex is repeated.
- A **cycle** is a closed path of length at least 1.

54. Eulerian graph

    A graph is **Eulerian** if it has a closed walk that contains every edges exactly once. **Eulerian circuit** or **Euler tour** in an undirected graph is a circuit that uses each edge exactly once.

55. De Bruijn Sequence

    A cylic sequence $(a_1, a_2, ..., a_{2^n})$ is a $(2, n)-$de Bruijn sequence if it satisfies the following conditions:

    (a) $a_i \in \{0, 1\}, i = 1, 2, ..., 2^n$; and

    (b) $(a_j, a_{j+1}, ..., a_{j+n-1}), j = 1, 2, ..., 2^n$, (mod 2^n), is a set of 2^n different $n-$dimension vectors.

56. $(2, n)$-De Bruijn digraph

    A $(2, n)$**-De Bruijn digraph** $D_{2,n}$ is a weighted digraph satisfying

    (a) $V(D_{2,n}) = (\mathbb{Z}_2)^{n-1}$; and

    (b) From $(a_1, a_2, \ldots, a_{n-1})$ (or simply $a_1a_2 \cdots a_{n-1}$) to $(a_2, a_3, \ldots, a_n)$ (or simply $a_2a_3 \cdots a_n$) is an arc with weight $a_1a_2 \cdots a_n$.

57. Postman tour

    A **postman tour** (or **covering walk**) is a closed directed walk that uses each arc at least once.

58. Optimal postman tour

    An **optimal postman tour** in a weighted graph, is a postman tour such that the sum of the weights of all the arcs is the minimum.

59. Hamiltonian Path, Hamiltonian Cycle and Hamiltonian graph

   - a **Hamiltonian path** (or traceable path) is a path in an undirected or directed graph that visits each vertex exactly once.
   - A **Hamiltonian cycle** (or Hamiltonian circuit) is a Hamiltonian path that is a cycle.
   - A graph that contains a Hamiltonian cycle is called a **Hamiltonian graph**.

60. Closure

   The (Hamiltonian) **closure** $C(G)$ of a graph G is the graph resulting from adding edges between non-adjacent vertices of degree sum $|G|$ until it is impossible to do so any further.

61. Independent set and Independence Number

   $G = (V, E)$ is a graph. $S \subseteq V$.

   - If no two distinct vertices in S of which are adjacent, then S is an **independent set** or **stable set**.
   - If S^* is a maximum independent set for G, then $\mid S^* \mid$ is called **independence number** of G, written $\beta(G)$

62. Power of Graph

   The **k-th power** G^k of an undirected graph G is a graph obtained from G that has the same set of vertices, but in which two vertices are adjacent when their distance in G is at most k. i.e. $E(G^k) = \{(u, v)|u, v \in V(G), 1 \leq d_G(u, v) \leq k\}$.

63. Difference

   For a vertex set $\mathbb{Z}_n$, the **difference** of i and j is $d(i, j) = \min\{|j - i|, n - |j - i|\}$.

64. Circulant graph

   Let $V(G) = \mathbb{Z}_n$ and D be a set of differences defined on $\mathbb{Z}_n$. Then, $G =_{def} C(n; D)$ is the graph $(\mathbb{Z}_n, E)$ such that $\{i, j\} \in E$ if $d(i, j) \in D$.

65. Girth

    The **girth** of a graph with a cycle is the length of its shortest cycle.

    An acyclic graph has infinite girth.

66. Connectivity

    The **connectivity** of a graph G, denoted $\kappa(G)$, is defined as

    $\kappa(G) = \min\{|S| | S \subseteq V(G)$ such that $G - S$ is a disconnected or has only one vertex$\}$.

67. Cut-edge and Cut-vertex

    A **cut-edge** (or **cut-vertex**) is an edge (or a vertex) whose removal increases the number of components.

68. Separating set and Vertex cut

    - A **separating set** or vertex cut of a graph G is a set $S \subseteq V(G)$ such that $G - S$ increases the number of components in G. ($c(G)$ denotes the number of components in G.)
    - A graph G is k**-connected** if its connectivity is at least k.

69. Edge-connectivity and k-edge-connected

    - The **edge-connectivity** of a connected graph G, denoted $\kappa'(G)$, is the minimum size of an edge subset F such that $G - F$ is not connected.
    - G is k**-edge-connected** $\kappa'(G) \geq k$.

70. Separating set, Vertex cut, Connectivity, and k-connected (Digraphs)

    - The **separating set** or **vertex cut** of a digraph D is a subset $S \subseteq V(D)$ such that $D - S$ is not strongly connected.
    - The **connectivity** of digraph D is
      $\kappa(D) = \min\{|S| | S \subseteq V(G), D - S$ is not strongly connected or contains only one vertex$\}$
    - D is a k-connected if the connectivity of D is at least k.

71. Edge cut (Digraphs)

An **edge cut** in a strongly connected digraph $G = (V, E)$ is an arc subset $F \subset E$ such that the arc-deletion subdigraph $G - F$ is not strongly connected.

72. Network, Flow, and Feasible flow

- A **network** is a digraph satisfying the following conditions:
  - A **source vertex** s and a **sink vertex** t are in the network.
  - A nonnegative **capacity function** $cap : E \to \mathbb{R}$ exists.
- Let $f : E(D) \to \mathbb{R}$ be given for a digraph D. The function f is called a **flow** if for every $v \in V(D)$, $\Sigma_{a\in E_v^+} f(a) = \Sigma_{a\in E_v^-} f(a)$.
- A **feasible flow** is a function $f : E \to \mathbb{R}$ which obeys the following constraints:
  - capacity constraints: $f(v, w) \leq \text{cap}(v, w)$, for each arc $(v, w) \in E$.
  - conservation constraints: $\Sigma_{(w,v)\in E} f(w, v) = \Sigma_{(v,w')\in E} f(v, w')$ for each vertex $v \in V - \{s, t\}$.
  - nonnegativity constrains: $f(v, w) \geq 0$, for each arc $(v, w) \in E$.
- Let e be an arc in a network, then e is **saturated** if $f(e) = cap(e)$, and e is **unsaturated** if $f(e) < cap(e)$.

73. Semipath, f-augmenting path, and Tolerance

Let a feasible flow f in a network N, and C is the capacity function.

- Any path in the underlying graph of N in G is called the **semipath** of N.
- Let P be a semipath which starts from source vertex s to sink vertex t, and for any $e \in E(P)$, if P is an f-**augmenting path** then the following two statements are true.
  - If the directions of P and arc e are respectively the same, then $f(e) < C(e)$.
  - If the directions of P and arc e are respectively different, then $f(e) > 0$.
- P is an f-augmenting path, the function ϵ is defined by the following:

$$\epsilon(e) = \begin{cases} C(e) - f(e), & \text{if the direction of } P \text{ and arc } e \text{ are the same.} \\ f(e), & \text{otherwise.} \end{cases}$$

  Then $min_{e\in E(P)}\epsilon(e)$ is the **tolerance** of P.

74. Net flow

    Let f be a flow in a network N.

    - Let $x \in V(N)$,
      $f^+(x) - f^-(x)$ is called **net flow leaving** x.
      $f^-(x) - f^+(x)$ is called **net flow into** x.
    - The net flow into sink vertex t is called the **value** of f, denoted $val(f)$.

75. Maximum flow

    The maximum feasible flow f is called **maximum flow**, and $val(f)$ is called maximum flow value.

76. Zero flow

    A flow f in a network N is called **zero flow** if $f(e) = 0$, $\forall e \in E(N)$.

77. Source set, Sink set, Source/Sink cut, and Capacity

    Let s be the source vertex, t be the sink vertex in a network N, and S and T be a partition of vertex set $V(N)$ which satisfy $s \in S$, $t \in T$, then S is a **source set**, and T is a **sink set**. Let $[S,T]$ be the set of arcs whose tails in S and heads in T. $[S,T]$ is called the **source/sink cut** in network N. If C is the capacity function of N, then the **capacity** of $[S,T]$ is defined by $cap(S,T) = \Sigma_{e \in [S,T]} C(e)$.

78. Vertex labeling, k-coloring, Colors, Color class, Proper, k-colorable, and Chromatic number

    - A **vertex labeling** is an assignment of vertices with labels in a graph G, typically $v_1, v_2, \ldots, v_n$ are assigned to the vertices of G respectively.
    - A k-**coloring** of a graph G is a function from its vertex-set $V(G)$ vertices to a set $C = \{1, 2, \ldots, k\}$ whose elements are called **colors**.
    - A **color class** for a graph with a coloring is the set of all vertices that are assigned the same color.
    - A k-coloring is **proper** if two adjacent vertices are always assigned different colors.
    - A graph is k-**colorable** if it has a proper k-coloring with k or fewer colors.

- The **chromatic number** of a graph G, denoted $\chi(G)$, is the smallest number k of colors such that G is k-colorable.

79. Proper subgraph

    H is a **proper subgraph** of graph G if $H \lneq G$.

80. k-chromatic, Optimal coloring, Color-critical, and k-critical

    - G is a k-**chromatic** if $\chi(G) = k$.
    - G is a **color-critical** or k-**critical** if $\chi(G) = k$ and for any H which is the proper subgraph of G, $\chi(H) < k$.

81. Outerplanar Graph

    An undirected graph is an **outerplanar graph** if it can be drawn in the plane without crossings in such a way that all of the vertices belong to the unbounded face of the drawing.

82. Perfect

    A graph is **perfect** if every induced subgraph has chromatic number equal to the size of maximum clique.

83. Intersection graph

    For each integer $p \geq 1$, the p-**intersection graph** of a family $\mathscr{F} = \{S_1, \ldots, S_n\}$ of subsets of a finite set S is defined to be the graph G having $V(G) = \mathscr{F}$ with $S_i S_j \in E(G)$ if and only if $i \neq j$ and $|S_i \cap S_j| \geq p$.

84. Intersection graph and Interval graph

    - The **intersection graph** $G(\mathscr{F})$ of a family $\mathscr{F}$ of subsets of a given set has as its vertices the members of $\mathscr{F}$; two vertices are adjacent if and only if the corresponding subsets of $\mathscr{F}$ have non-empty intersection.
    - A graph G is an **interval graph** if it is isomorphic to the intersection graph of a family of intervals of a line.

85. Chordal graph

    A **chordal graph** is one in which all cycles of four or more vertices have a chord, which is an edge that is not part of the cycle but connects two vertices of the cycle.

86. Subdivision

    A **subdivision** of edge $e = uv$ in graph G is the graph obtained from G by replaying e by the path $< u, w, v >$ where w is a new vertex of degree two.

87. H-subdivision

    The graph H' is a subdivision of H, if one can obtain H' from H by a series of edge subdivisions.

88. Chromatic Polynomial

    The **chromatic polynomial** $P(G, \lambda)$, $\lambda \in \mathbb{N}$, of graph $G = (V, E)$ is the function whose value at λ ($\lambda = 1, 2, 3, \ldots$) is the number of proper colorings $\varphi : V \to \{1, \ldots, \lambda\}$ of G with at most λ colors. Here, two colorings are counted as different even if they yield the same color class by renumbering the colors.

89. Overfull

    An **overfull** graph G is a graph whose size is greater than the product of its maximum degree and half of its order floored, i.e. $|E(G)| > \Delta(G)\lfloor\frac{|V(G)|}{2}\rfloor$.

90. Class 1 and Class 2

    - A simple graph G is **Class** 1 if $\chi'(G) = \Delta(G)$.
    - A simple graph G is **Class** 2 if $\chi'(G) = \Delta(G) + 1$. ($\chi'(G) \geq \Delta(G) + 1$ for multigraphs.)

91. Matching, M-saturated and Perfect matching

    - A **matching** in $G = (V, E)$ is a set $M \subseteq E$ of pairwise nonadjacent edges.
    - A vertex v is **M-saturated** if v is incident to an edge of M.
    - A **perfect matching** in $G = (V, E)$ is a matching M in which each vertex of V is incident on exactly one edge of M.

92. Maximal matching and Maximum matching

- M is a **maximal matching** if it is not a proper subset of any other matching in graph G. In other words, a matching M of a graph G is a maximal matching if every edge in G has a non-empty intersection with at least one edge in M.
- A **maximum matching** of G is a matching M having the largest size.

93. Total coloring, Total chromatic number, k-total coloring and k-total colorable

- **Total coloring** is a type of graph coloring on **the vertices and edges of a graph**. When used without any qualification, a total coloring is always assumed to be proper in the sense that no adjacent edges and no edge and its endvertices are assigned the same color.
- The **total chromatic number** $\chi''(G)$ of a graph G is the least number of colors needed in any total coloring of G.
- A graph G is called k**-total coloring** if the total coloring number is k.
- A graph G is called k**-total colorable** if G has a k-total coloring.

94. Type I and Type II

G is called **Type I** if $\chi''(G) = \Delta(G) + 1$, otherwise G is called **Type II**.

95. Biconformability

Given an equibipartite graph G and given a vertex-colouring which assigns the colours $c_1, c_2, \ldots, c_{\Delta(G)+1}$, let A_i be the set of vertices of A coloured c_i and B_i the set of vertices of B coloured c_i. Let $a_i = |A|$ and $b_i = |B|$. If W is a subset of $V(G)$, let $V_{<\Delta}(W)$ denote the set of vertices in W which have degree less than Δ in the graph G. Call G **biconformable** if G has a vertex-colouring such that for $1 \le i \le \Delta(G) + 1$,

$$|V_{<\Delta}(A \setminus A_i)| \ge b_i - a_i,$$

$$|V_{<\Delta}(B \setminus B_i)| \ge a_i - b_i,$$

and

$$\mathrm{def}(G) \ge \sum_{i=1}^{\Delta(G)+1} |a_i - b_i|.$$

96. List coloring and Choice number

- A (vertex) **list assignment** L on a graph G associates a set L_v of colors with each vertex v of G. Each L_v is interpreted as the set of followed colors for vertex v.
- The graph G is **L-colorable** (or **list colorable**, when L is understood from context) if it admits a proper vertex-coloring φ such that $\varphi(v) \in L_v$ for all v.
- If $|L_v| = k$ for all $v \in V$, then the list assignment L is called a $k-$**assignment** L.
- The **choice number** of G, denoted $ch(G)$, is the smallest nonnegative integer k such that G is $k-$choosable. (In part of the literature, the choice number is called *list chromatic number*, and also the notation $\chi_l(G)$ is commonly used for $ch(G)$.)

97. Edge choice number

The **edge choice number** (or **list chromatic index** or **list edge chromatic number**) is the minimum list-size that guarantees a list edge-coloring of G; it is denoted by $ch'G$ (or by $\chi'_l(G)$)

98. Edge labeling, k-edge-coloring, Proper, k-edge-colorable and Chromatic index

- An **edge labeling** is a function mapping E to a set of labels.
- A **k-edge-coloring** is a function mapping E to a k-set $\{1, 2, \ldots, k\}$ of labels. The labels $1, 2, \ldots, k$ is called **color**. A subset assigned to the same color is called a **color class**.
- A k-edge-coloring of a graph is almost always a **proper** coloring, namely a labelling of the edges with colors such that no two edges sharing the same vertex have the same color.
- A graph that can be assigned a (proper) k-edge-coloring is **k-edge-colorable**
- The smallest number of colors needed for an edge-coloring of a graph G is the **chromatic index**, or edge chromatic number, $\chi'(G)$.

99. Embeddable graphs

A (p, q) graph G is said to be **embeddable** on a surface S if it is possible to distinguish a collection of p points of S that correspond to the vertices of G and a collection of q curves (closed), pairwise disjoint except possibly for endpoints, on S that correspond to the edges of G such that if a curve A corresponds to the edge $e = uv$, then only the endpoints of A correspond to u and v.

100. Planar graphs

A graph is **planar** if it can be embedded in the plane and equivalently on a sphere.

101. Proper drawing of graphs on a surface

A drawing is **proper** if the follows satisfy :

(a) adjacent edges never cross,

(b) two nonadjacent edges cross at most once,

(c) no edge crosses itself,

(d) no more than two edges cross at a point of the plane, and

(e) the (open) arc in the plane corresponding to an edge of the graph contains no vertex of the graph.

102. Crossing number

The **crossing number** $v(G)$ of a graph is the minimum number of crossings (of its edges) among the drawings of G in the plane.

103. Surface

A **surface** (in $\mathbb{R}^3$) is a compact orientable 2-manifold that may be thought of as a sphere on which has been placed a number of **handles** or equivalently, a sphere in which has been inserted a number of **holes**.

104. Orientable genus

The number of handles (or holes) is referred to as the **genus of the surface**. The **genus (orientable) of a graph** G, $\gamma(G)$, is meant the smallest genus of all surfaces on which G can be embedded.

105. 2-cell embedding

A region is called a **2-cell** if any simple closed curve in that region can be continuously deformed or contracted in that region to a single point. An embedding of a graph G on a surface S is called a **2-cell embedding** of G on S if all regions so determined are 2-cells.

106. Non-orientable surfaces

A **crosscap** is obtained by attaching the boundary of a Möbius band to a cycle on a sphere. A **non-orientable surface** N_k can be recognized as placing k crosscaps on a sphere, $k \geq 0$.

107. Maximum genus

Let G be a connected graph. The **maximum genus** $\gamma_M(G)$ of G is the maximum among the genera of all surfaces on which G can be 2-cell embedded.

108. Random graph

- Let $p(n)$ be a number in $[0, 1]$ and n is the number of vertices of a graph G. Then, $G(n, p(n))$ is a **random graph** generated by the set of edges each (independently) has probability $p(n)$.
- If G_0 is of order n and size m, then the event $\{G_0\}$ is of probability $p^m \cdot (1-p)^{\binom{n}{2}-m}$ where $p \approx_{def} p(n)$. If $p(n)$ is a constant, we simply write p instead of $p(n)$.