離散數學期末考 2018, 6, 15; 10:10-12:30

- Explain your answer, as clear as possible.

1. Give the definitions of the following terms. (36 points)

   (a) Partial ordered set

   (b) Critical set (Latin square)

   (c) Graph radius

   (d) Planar graph

   (e) Eulerian circuit

   (f) Incidence matrix (of a design)

2. There is a necklace with 8 colored stones. (16 points)

   (a) If every stone can be either blue or red, then how many different patterns can we have? (Consider rotation and reflection.)

   (b) If there are six blue stones and two red stones, then how many different patterns can we have? (Consider rotation and reflection.)

3. Let $S = \{1, 2, \cdots, 11\}$. Find the maximum number of subsets of $S$ such that no subset in $S$ is contained in any other subset in $S$. (兩兩互相不包含) (12 points)

4. Find twelve 3-subsets of $\{1, 2, \cdots, 9\}$ such that any two of them contain at most one common element. (12 points)

5. Construct four mutually orthogonal Latin squares of order 5. (Hint: Use $< \mathbb{Z}_5, + >$ and $< \mathbb{Z}_5^\star, \cdot >$) (12 points)

6. Find a graph $G$ of order 10 such that every vertex of $G$ is of degree 3 and the diameter of $G$ is 2. (12 points)

(背面尚有試題)

(Bonus Parts)

7. Prove one of the following two statements for 12 points.

   (a) If $G$ is a connected planar graph with $p$ vertices, $q$ edges and $r$ regions, then $p - q + r = 2$.

   (b) If $G$ is a tree, then $G$ has exactly $p - 1$ edges where $p$ is the number of vertices in $G$.

8. Prove one of the following two statements for 12 points.

   (a) For each positive integer $n$, $\dfrac{n^8 + 17n^4 + 6n^2}{24}$ is an integer.

   (b) Let $L$ be the Latin square obtained from the multiplication table of $< \mathbb{Z}_n, + >$, see Figure 1 for an example. Prove that if $n$ is even, then $L$ does not contain any Latin transversal, i.e., we can not find $n$ distinct elements from distinct rows and columns.

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 | 0 |
| 2 | 3 | 4 | 5 | 0 | 1 |
| 3 | 4 | 5 | 0 | 1 | 2 |
| 4 | 5 | 0 | 1 | 2 | 3 |
| 5 | 0 | 1 | 2 | 3 | 4 |

Figure 1. $n = 6$.