Lecture 1 Feb. 23

Sets, Relations (離散數學).

A : Set (無定義名詞), $x \in A$ if x is in the set A (x is an element of A)
$x \notin A$ if x is not an element of A

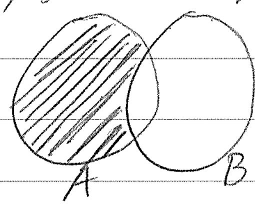
$|A|$: Cardinality of A (the number of elements in A).

(1) $|A| = 0$ if and only if (iff) $A = \phi$.

(2) A, B are sets, $A \subseteq B$ iff $\forall x \in A$, $x \in B$.
$A \nsubseteq B$ iff $\exists x \in A$ and $x \notin B$.

(3) $\phi \subseteq A$. (3′) $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$, (3″) $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

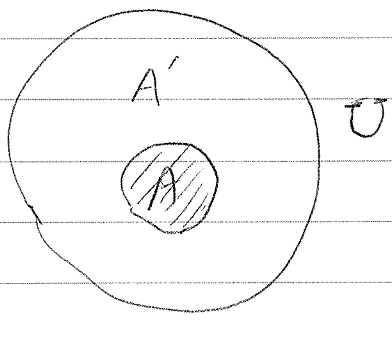
(4) $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$.

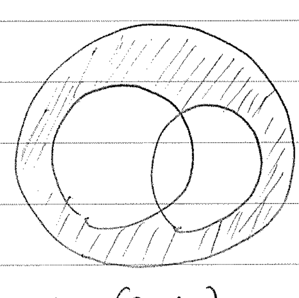


$A \setminus B$.

(5) $A \triangle B = (A \setminus B) \cup (B \setminus A)$.

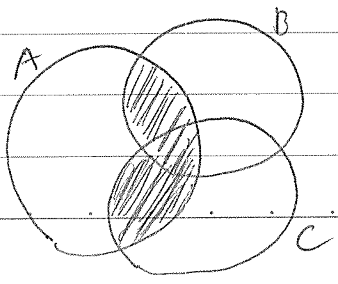
(6) If $A \subseteq U$, then $A' = \{x \in U \text{ and } x \notin A\}$.

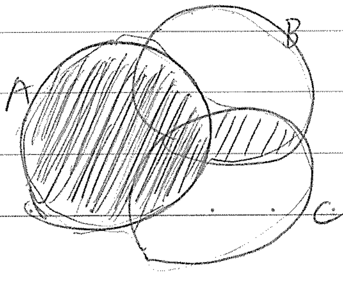




(7) $(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$.

(8) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.





(9) $|A| = |B|$ iff $\exists\ \varphi$ (bijection), $\varphi : A \to B$.

$|A| \le |B|$ iff $\exists$ 1-1 mapping π, $\pi : A \to B$

$|A| \ge |B|$ onto mapping!

(10) If $|A| = n$ is finite, then $\forall\ C \subsetneq A$, $|C| < n$.
(A is an n-set.)

(11) If $|A|$ is infinite, then there exists a proper subset of A, say $\tilde{A} \subsetneq A$, such that $|A| = |\tilde{A}|$.

(12) Even A_1 and A_2 are both infinite sets, $|A_1|$ may not be equal to $|A_2|$. $(|\mathbb{N}| < |\mathbb{R}|)$

(13) If A is a finite set, then $2^A = \{X \mid X \subseteq A\}$ is called the powerset of A and $|2^A| = 2^{|A|}$. Also, $\mathbb{P}(A) = 2^A$.

(14) $A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$.

Problem 1 (Bonus) Find the maximum number of proper subsets of an n-set such that no two of them are contained each other.

(•) Note that if $\mathbb{B} \subseteq 2^X$, then $(X, \mathbb{B})$ is called a $\boxed{\text{design}}$

(15) A relation "$\sim$" between two sets A and B is a set of ordered pairs $(a, b) \in A \times B$, denoted by $a \sim b$.

(16) If $A = B$, then we simply say "$\sim$" is a relation of A. (between the elements of A)

(17) Let "$\sim$" be a relation.

(i) $\sim$ is reflexive if $a \sim a \ \forall a \in A$.

(ii) $\sim$ is symmetric if $a \sim b \Rightarrow b \sim a$.

(iii) $\sim$ is transitive if $a \sim b, b \sim c \Rightarrow a \sim c$.

(18) A relation of A satisfying (i), (ii) and (iii) is an <u>equivalence relation</u>.

(19) Let $\sim$ be an equivalence relation of A. Then, A can be partitioned into subsets (equivalence classes), denoted by $A/\sim$.

(20) If $A_1, A_2, \cdots, A_t$ are equivalence classes, then $A_i \cap A_j = \phi$ and $\bigcup_{i=1}^{t} A_i = A$.

<u>Example</u> $\mathbb{Z}$ can be partitioned into 7 subsets (equivalence classes) by using the relation, $a \sim b$ iff $a \equiv b \pmod 7$.

$\mathbb{Z} = \bigcup_{i=0}^{6} A_i$, $A_i = \{ x \overset{\in \mathbb{Z}}{} \mid x \equiv i \pmod 7 \}$.