Oct. 13

$(0,1,2^+)$-model for 2-defective case

Fact 1 $M_{2^+}(2,n) \geq \left\lceil \log_2 \binom{n}{2} \right\rceil$. (In worst case!)

Fact 2 Binary splitting algorithm works!

e.g. $d=2$ and $n=12$. $D=\{3,9\}$

1 2 3 4 5 6 (1)

1 2 3 4 5 6 7 8 9 (2)

1 2 3 4 5 6 8 9 (2)

1 2 3 4 5 6 8 (1)

$\leftarrow 9 \in D$

1 2 3 9 (2)

2 3 9 (2)

2 9 (1)

$\leftarrow 3 \in D$.

$D = \{7, 11\}$

1 2 3 4 5 6	(0)	11, 7, 8	(2)
7 8 9	(1)	11, 7	(2)
7 8 9 10 11	(2)	Done!	
7 8 9 10	(1)		

$\leftarrow 11 \in D$.

Fact 3 In general, $(0, 1, 2, \cdots, d^+)$-model is easier to find the set of positive items (in complexity). We expect to find an algorithm which can attain the lower bound $\lceil \log_{d+1} \binom{n}{d} \rceil$. (Can this be done? Ex. 4)
(No!!)

(*) This model is very closed to find an edge in complete graph. But, the "query" is slightly different.

Let G be a complete graph of order n, i.e., $G \cong K_n$.

Q : Does a set $S \subseteq V(G)$ contain the hidden edge e, i.e. $e \in \langle S \rangle_G$?

(**) The difference is on the case "1". If $e = \{x, y\}$ and $x \in S$ but not y, then $e \notin \langle S \rangle_G$ and the answer is simply "no". (Not one vertex!)

Fact 4 The splitting algorithm used in $(0, 1, 2^+)$-model is also good to find an edge.

$V(G) = \{1, 2, \cdots, 12\}$, $e = \{3, 10\}$

Q_1 : $S = \{1,2,3,4,5,6\}$ No.

Q_2 : $S = \{7,8,9,10,11,12\}$ No. ← Extra

Q_3 : $S = \{1,2,3,4,5,6\} \cup \{7,8,9\}$ No.

Q_4 : $S = \{1,2,3,4,5,6,10,11\}$ Yes

Q_5 : $S = \{1,2,3,4,5,6,10\}$ Yes

Q_6 : $S = \{1,2,3,10\}$ Yes.

Q_7 : $S = \{1,2,10\}$ No $\longrightarrow \{3,10\}$.

(In case Q_7 asks $S = \{2,3,10\}$, yes and one more test is necessary.)

$=$

$Q_3 - Q_7$ are trying to find an edge in $K_{6,6}$

So, in the worst case, we need $\lceil \log_2 36 \rceil$ tests to find the hidden edge.

<u>Problem 1</u> Find a hidden subgraph of K_n.

<u>Problem 2</u> Find a hidden edge in a bipartite graph.

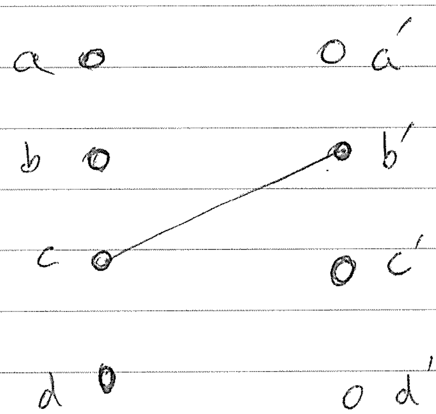
Remark The best result obtained so far is the following.

Theorem Let H be a hidden subgraph in K_n where $\|H\| = m$. Then, H can be determined by an algorithm which uses at most $m\log_2 n + 10m + 3n$ detecting queries.
(The informatic lower bound is $\left\lceil \log_2\left(\sum_{i=0}^{m} \binom{\binom{n}{2}}{i} \right) \right\rceil$.)

$n = 12$

So, if $m=1$, then $\left\lceil \log_2\left(\binom{66}{0} + \binom{66}{1} \right) \right\rceil = 7$.
(The above algorithm uses "$\lceil \log_2 12 \rceil + 10 + 36$" queries, but the work is good when n is getting larger.)

As to problem 2, there are interesting facts inside.



(*) It takes 4 queries to find an edge in $K_{4,4}$.
(How?)

The reason that (4) queries are enough is due to the fact:

we can always find a subgraph of size (half).
(induced)

<u>Conjecture</u> Given a bipartite graph of size 2^m. Then, there exists an induced subgraph of size 2^{m-1} for each $m \in \mathbb{N}$

<u>Remark</u> This conjecture was posed by G.J. Chang and F.K. Hwang in an early paper. But, this conjecture was popularized by P. Erdös later. It is known as one of <u>Erdös' conjectures</u> with prize. (100,- U.S. dollars ?).

The best result known so far shows that this conjecture is true for $m \le 5$. (阮夙姿與張鎮華)

(**) You are encouraged to work on this problem for your Thesis

<u>Problem 2'</u> Can we replace "bipartite" with "tripartite" ?

<u>Problem 2''</u> Can we replace 2^m with <u>$2m$</u> and find an induced subgraph of size m ? (In general, No!)

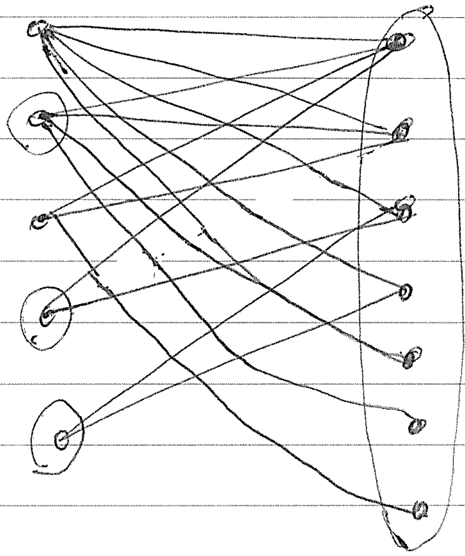
Observations

Let G be a bipartite graph with bi-degree sequence $\langle d_1, d_2, \cdots, d_h ; c_1, c_2, \cdots, c_k \rangle$. ($G = \langle A, B \rangle$, $|A| = h$, $|B| = k$ and $\|G\| = 2^m$

① $\sum_{i=1}^{h} d_i = \sum_{j=1}^{k} c_j = 2^m$. $\quad (d_1 \geq d_2 \geq \cdots \geq d_h ; c_1 \geq c_2 \geq \cdots \geq c_k)$.

② If there exists a sub-sequence of $\langle d_1, d_2, \cdots, d_h \rangle$ (resp. $\langle c_1, c_2, \cdots, c_k$ such that their sum is 2^{m-1}, then we are done.

$$\langle 5, 4, 3, 2, 2 ; 4, 4, 3, 2, 1, 1, 1 \rangle$$



③ So, what's left to consider? How about the following?

$$\langle 3, 3, 3, 3, 3, 1 ; 5, 5, 5, 1 \rangle$$