Definition 3

1. Drawing and Crossing

   - In a **drawing graph**, vertices are represented by points and edges are represented by curves such that any two edges intersect at most in a finite number of points.
   - A **crossing** is a point of a graph drawing where two edges intersect.

2. Planar graph and Planar embedding

   A **planar graph** is a graph which can be drawn on a sphere(plane) in such a way that no edges cross each other. And it is also called **planar embedding** of the graph.

3. Open set, Region, Face and Plane

   - A subset U of a metric space (M, d) is called **open** if, given any point $x \in U$, there exists a real number $\varepsilon > 0$ such that, given any point y in M with $d(x, y) < \varepsilon$, y also belongs to U. Equivalently, U is open if every point in U has a neighbourhood contained in U.
   - A **region** of a planar graph is defined to be an area of the plane that is bounded be edges and is not further divided into subareas.
   - A **face** is a region of a planar drawing, and the unbounded region is called the external face, if the drawing is on a plane.
   - A plane graph is a graph which is drawn on a plane.

4. Dual graph

   The **dual graph** of a plane graph G is a graph that has a vertex for each face of G and two vertices are adjacent if and only if their corresponding faces of G are separated from each other by an edge. Thus, each edge e of G has a corresponding dual edge, the edge that connects the two faces on either side of e.

5. Maximal Planar Graph

   A planar graph G is said to be triangulated (also called **maximal planar**) if the addition of any edge(not a multiple edge) to G results in a nonplanar graph.

6. Maximal Outerplanar Graph

   A **maximal outerplanar graph** is an outerplanar graph that cannot have any additional edges added to it while preserving outerplanarity.

7. Minimal Non-planar Graph

   A **minimal non-planar graph** is not planar, but every proper subgraph is planar.

8. Convex Embedding

   A **convex embedding** of a graph is a planar embedding in which each face boundary is a convex polygon.

9. Thickness

   The **thickness** of a graph G is the minimum number of planar graphs into which the edges of G can be partitioned.

10. Crossing Number

    The **crossing number** $cr(G)$ of a graph G is the smallest(minimum) number of edge crossings of a plane drawing of the graph G.

11. Proper face coloring, Cubic graph and Tait coloring

    - A **proper face coloring** of a planar graph assigns a color to each face or region so that no two faces that share a boundary have the same color.
    - A **cubic graph** is a 3-regular graph.
    - A **Tait coloring** is a 3-edge coloring of a cubic graph.

12. Graceful labeling and graceful graph

    A **graceful labeling** of a graph with m edges is a labeling of its vertices with some subset of the integers between 0 and m inclusive, such that no two vertices share a label, and such that each edge is uniquely identified by the positive, or absolute difference between its endpoints. A graph which admits a graceful labeling is called a **graceful graph**.

13. Harmonious labelling and Harmonious graph

A **harmonious labelling** on a graph G is an injection from the vertices of G to the group of integers modulo k, where k is the number of edges of G, that induces a bijection between the edges of G and the numbers modulo k by taking the edge label for an edge (x, y) to be the sum of the labels of the two vertices $x, y \pmod{k}$. A **harmonious graph** is one that has a harmonious labelling.

14. Prime labelling

    A **prime labelling** of a graph is an injective function $f : V(G) \to \{1, 2, ..., |V(G)|\}$ such that for every pair of adjacent vertices u and v, $\gcd(f(u), f(v)) = 1$(labels of any two adjacent vertices are relatively prime).

15. Magic graph and Supermagic

    A **magic graph** is a graph whose edges are labelled by positive integers, so that the sum over the edges incident with any vertex is the same, independent of the choice of vertex; or it is a graph that has such a labelling. If the integers(used in labelling) are the first q positive integers, where q is the number of edges, the graph and the labelling are called **supermagic**.

16. Random graph Model

    Different random graph models produce different probability distributions on graphs.

    - Most commonly studied is the $G(n, p)$, in which every possible edge occurs independently with probability $0 < p < 1$. The probability of obtaining any one particular random graph with m edges is $p^m(1-p)^{N-m}$ with the notation $N = \binom{n}{2}$.
    - A closely related model, the Erdös - Rényi model denoted $G(n, M)$, assigns equal probability to all graphs with exactly M edges. With $0 \leq M \leq N$, $G(n, M)$ has $\binom{N}{M}$ elements and every element occurs with probability $\binom{N}{M}^{-1}$.

17. Dominating set

    A set V is a **dominating set** of a graph $G = (V, E)$ if each vertex in V is either in S or is adjacent to a vertex in S. And the **dominating number** $\gamma(G)$ is the minimum cardinality of a dominating set of G.

18. Connected dominating set, Independent dominating set and Total dominating set

    Let S be a dominating set of a graph G.

- S is a **connected dominating set** if the induced subgraph $G[S]$ of S is connected.
- S is an **independent dominating set** if the induced subgraph $G[S]$ of S is independent.
- S is a **total dominating set** if there don't exist isolation vertices in the induced subgraph $G[S]$ of S.