

1. Prove that if G is not an odd cycle or a complete graph, then $\Delta(G) \geq \chi(G)$.
2. If G is a bipartite graph, prove that G is of Class 1.
3. Let G be a complete bipartite graph. Prove that G is of type 1 if and only if G is not a regular graph.
4. Explain the discharging method by using two examples.
5. Prove that there exists a C_4 – free graph whose chromatic number is larger than 100.
6. Let $\gamma(G)$ denote the genus of a graph G . Find $\gamma(K_{101})$.
7. Prove that if G is of genus k , then $p - q + r = 2 - 2k$ where p , q and r are the number vertices, edges and regions of G .
8. Prove or disprove that there exists a 2-cell embedding of K_6 on a surface of genus 3.
9. Let $cr(G)$ denote the crossing number of G . Find $cr(K_8)$.
10. Find a graph G with as large as large order as possible such that both G and its complement, \overline{G} , contain no subgraph K_4 .