Idea of finding $\chi''(G)$.

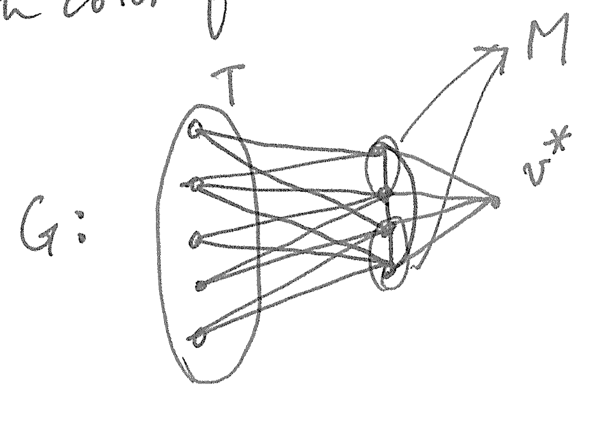
<u>Step 1.</u> Find an independent set T such that T is of large size.

<u>Step 2.</u> Constructing a graph G^* from G by letting $V(G^*) = V(G-T) \cup \{v^*\}$ and $E(G^*) = E(G) \cup \{v^*u \mid u \in V(G-T)\} - M$ where M is a matching in $\langle G-T \rangle_G$.

(M should be chosen such that $\Delta(G-M)$ is as small as possible.)

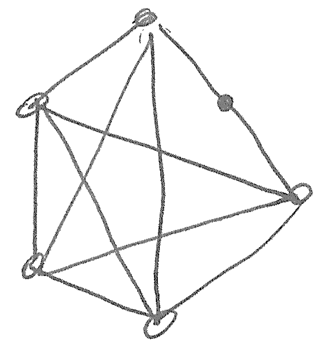
<u>Step 3.</u> $\chi''(G) = \chi'(G^*) + 1$.

Color $T \cup M$ with one color and the vertex u with color of v^*u in the edge coloring of G^*.

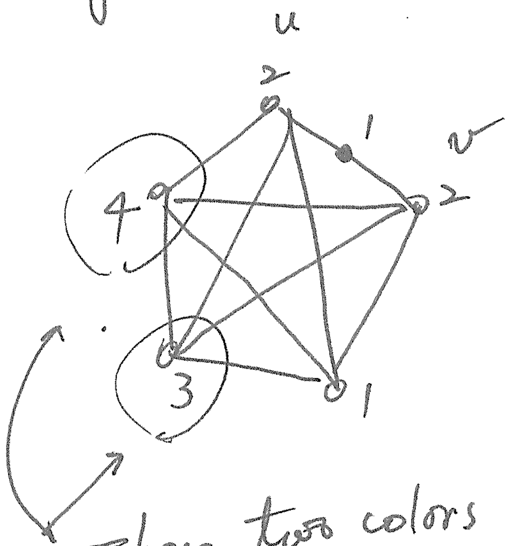


<u>e.g.</u> (About conformable idea)

G:



It is conformable since we have a vertex coloring φ satisfying the condition.



These two colors provide different parity with the order "6".

If $\varphi(u) \neq \varphi(v)$, then clearly G is of Type 2.

So, it suffices to consiter $\varphi(u) = \varphi(v)$.

Again, we count the possible occurrence of "5" colors (if G is of type 1).

1 : 4 times

2 : 3 times

3 : 3 times

4 : 3 times

5 : 3 times

In total, at most 16 occurrence of elements in $V(G) \cup E(G)$.

But, there are 17 elements (6 vertices and 11 edges) to color. Hence, G is of type 2.

There are two conjectures to prove:

1. Total Coloring Conjecture

$$\chi''(G) \leq \Delta(G) + 2$$

2. Conformable complete multipartite graphs Conjecture

A complete multipartite graph G is of type 1 if and only if G is conformable.

(It has been proved that if $|G|$ is odd, then the conjecture holds.)

Also, if the graph has only two partite sets, then the conjecture is also true.

(*) A complete bipartite graph $K_{m,n}$ is conformable if and only if $m \neq n$.