1. Graph

   A **graph** $G = (V, E)$ consists of two sets V and E.

   - The elements of V are called **vertices** (or **nodes**).
   - The elements of E are called **edges**.
   - Each edges has a set of one or two vertices associated to it, which are called its **endpoints**. An edge is said to **join** its endpoints.

2. Hypergraph

   A **hypergraph** $H = (X, F)$ has vertex set X; its edge set F consists of subsets of X, i.e. $F \subseteq 2^X$.

3. Edge-types

   - A **loop** is an edge that joins a single endpoint to itself.
   - A **multiple edge** is a collection of two or more edges having identical endpoints.

4. Simple graph

   A **simple graph** is a graph that has no self-loops or multi-edges.

5. Multigraph and Pseudograph

   - A **multigraph** is a graph which is permitted to have multiple edges, that is, edges that have the same endpoints.
   - A **pseudograph** is a non-simple graph in which both graph loops and multiple edges are permitted.

6. Finite and Infinite graph

   A **finite graph** is a graph with a finite vertices of points and edges. A graph which is not finite is called **infinite**.

7. Neighbors

   A vertex u is **adjacent** to vertex v if they are joined by an edge. Two adjacent vertices may be called **neighbors**. And v (or u) is said to be **incident** on the edge.

8. Null graph

   A **null graph** is a graph whose vertex- and edge-sets are empty.

9. Complete graph

   A **complete graph** is a graph in which every pair of vertices is joined by an edge.

10. Partite sets

    - A simple graph or multigraph is **bipartite** if its vertices can be partitioned into two sets(called **partite sets**) in such a way, that no edge joins two vertices in the same set.
    - A **complete bipartite graph** is a simple bipartite graph in which each vertex in one partite sets is adjacent to all the vertices in the other partite set.

11. Independent set

    $G = (V, E)$ is a graph. $S \subseteq V$

    - If every two distinct vertices in S are adjacent, then S is a **clique**.
    - If no two distinct vertices in S of which are adjacent, then S is an **independent set** or **stable set**.
    - If S^* is a maximum independent set for G, then $\mid S^* \mid$ is called **independent number**, written $\beta(G)$

12. Union and Intersection

    Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs.

    - $G = (V, E)$ is the **union** of G_1 and G_2, if $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$.
    - $G = (V, E)$ is the **intersection** of G_1 and G_2, if $V = V_1 \cap V_2$ and $E = E_1 \cap E_2$.

13. Decomposition

    If all the following are satisfy, then we said G_1,G_2,$\cdots$,G_t are the **decomposition** of G.

    - For all $i \in 1, 2, \cdots, t$, $V(G_i) \subseteq V(G)$.
    - $E(G_1) \cup E(G_2) \cup \cdots \cup E(G_t) = E(G)$.
    - For any $i \neq j$, $1 \leq i, j \leq t$, $E(G_i) \cap E(G_j) = \emptyset$.

14. Product

The **cartesian product** (or **product**) of two graphs G and H is denoted by $G \times H$, where

$$V(G \times H) = V(G) \times V(H) \text{ and}$$
$$E(G \times H) = E(G) \times V(H) \cup V(G) \times E(H).$$

The endpoints of the edge $(d, v) \in E(G) \times V(H)$ are the vertices (x, v) and (y, v), where x and y are the endpoints of edge $d \in E(G)$.

The endpoints of the edge $(u, e) \in V(G) \times E(H)$ are the vertices (u, s) and (u, t), where s and t are the endpoints of edge $e \in E(H)$.

15. Join

The **join** (or **suspension**) of two graphs G and H is denoted by $G + H$, where

$$V(G + H) = V(G) \cup V(H) \text{ and}$$
$$E(G + H) = E(G) \cup E(H) \cup \{uv | u \in V(G) \text{ and } v \in V(E)\}.$$

16. Composition

The **composition** $G[H]$ of a graph G with a graph H is the graph with vertex set $V(G) \times V(H)$ such that (u_1, v_1) is adjacent to (u_2, v_2) whenever either u_1 is adjacent to u_2, or v_1 is adjacent to v_2 with $u_1 = u_2$.

17. Complement

The **complement**(or **edge-complement**) $G^c = (V, E^c)$ of a simple graph $G = (V, E)$ has the same vertex set V as G and edges defined : (x, y) is in E^c if and only if (x, y) is not in E.

18. Line graph

The **line graph** $L(G)$ of a graph G has the edges of G as its vertices; two vertices of $L(G)$ are adjacent if the edges in G to which they correspond have a common vertex. Also, a graph H is said to be a **line graph** if there exists a graph G such that H is isomorphic to $L(G)$.

19. Edge-contraction

The operation of **edge-contraction** produces a graph with edge-set $E - \{e\}$ but with a vertex set obtained by replacing ("merging") the vertices defining e in G , thus creating a new single vertex where the latter inherits all of the adjacencies of the pair of replaced vertices, without introducing loops or multiple edges.

20. Contraction graph

    H is a **contraction graph** of G if we contraction some edges in G.

21. Vertex splitting

    A **vertex splitting** is the inverse operation of an edge contraction.

22. Split graph

    A graph is a **complete split graph** if it can be partitioned in an independent set and a clique such every vertex in the independent set is adjacent to every vertex in the clique.

23. Subgraph and Supergraph

    A graph H is called a **subgraph** of graph $G = (V, E)$ if $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$, and $V(H)$ contains all the endpoints of all the edges in $E(H)$. And G is a **supergraph** of H.

24. Edge-deletion

    $G = (V, E)$ is a graph, $e \in E$, then $G - e$ is defined by $G - e = (V, E \setminus e)$. If $T \subseteq E$, then $G - T$ is defined by $G - T = (V, E \setminus T)$.

25. Vertex deletion

    A graph $G = (V, E)$, $v \in V$, then $G - v$ is defined by $G - v = (V \setminus v, E')$, where $E' = \{e | e \in E$ and v is not the endpoint of $e\}$. If $S \subseteq V$, then $G - S$ is defined by $G - S = (V \setminus S, E')$, where $E' = \{e | e \in E$ and e has no endpoints in $S\}$.

26. Induced subgraph

    In graph G, the **induced subgraph** on a set of vertices $W = \{w_1, \ldots, w_k\}$, denoted $G(W)$, has W as its vertex-set, and it contains every edge of G whose endpoints are in W. That is, $V(G(W)) = W$ and $E(G(W)) = \{e \in E(G) |$ the endpoints of edge e are in $W\}$

27. Edge induced subgraph

    Each subset $E' \subseteq E$ defines a unique subgraph $G' = (V', E')$ of graph $G = (V, E)$, where V' consists of only those vertices which are the endpoints of the edges in E'. The subgraph G' is called the **induced subgraph of G on the edge set** E'. Note that an edge-induced subgraph will not have isolated vertices.

28. Neighborhood

The **neighborhood** of a vertex v of a graph is the set of all vertices adjacent to v. It is denoted by $N(v)$.

29. Degree

The **degree** (or **valence**) of a vertex v in a graph G, denoted $deg(v)$, is the number of proper edges incident on v plus twice the number of self-loops. (For simple graphs, of course, the degree is simply the number of neighbors.) The **maximum degree** of graph G is $\triangle(G) = max\{deg(v) \mid v \in V(G)\}$ and the **minimum degree** of G is $\delta(G) = min\{deg(v)|v \in V(G)\}$

30. Order and Size

A graph $G = (V, E)$, the **order** of G is $|V|$. The **size** of G is $|E|$. And the **volume** of G is $\sum_{v \in V(G)} deg(v)$.

31. Regular graph

A graph is **regular** if every vertex is of the same degree.

It is **k-regular** if every vertex is of degree k. And k is called **valency** of G.

32. Isolated vertex

An **isolated vertex** in a graph is a vertex of degree 0.

33. Degree sequence

The **degree sequence** of a graph G is the sequence of its degrees of vertices values,arranged from the largest value to the smallest.

34. Adjacency matrix and Incidence matrix

- An **adjacency matrix** for a simple graph G whose vertices are explicitly ordered $v_1, v_2, \cdots, v_n$ is the $n \times n$ matrix A_G such that

$$A_G(i, j) = \begin{cases} 1, & \text{if } v_i \text{ and } v_j \text{ are adjacent; and} \\ 0, & \text{otherwise.} \end{cases}$$

- An **incidence matrix** for a simple graph G whose vertices are explicitly ordered $v_1, v_2, \cdots, v_n$ and edges are explicitly ordered $e_1, e_2, \cdots, e_m$ is the $n \times m$ matrix B_G such that

$$B_G(i,j) = \begin{cases} 1, & \text{if } v_i \text{ and } e_j \text{ are incident; and} \\ 0, & \text{otherwise.} \end{cases}$$

35. Isomorphic

An **isomorphism between two simple graphs** G and H is a vertex bijection $\phi : V_G \to V_H$ such that for $u, v \in V_G$, the vertex u is adjacent to the vertex v in graph G if and only if $\phi(u)$ is adjacent to $\phi(v)$ in graph H. Implicitly, there is also an edge bijection $E_G \to E_H$ such that $uv \mapsto \phi(u)\phi(v)$. We say that G and H are **isomorphic graphs** and we write $G \cong H$.

36. Relation and Equivalence relation

A **relation** R on a finite set S is a subset of the cartesian product $S \times S$.

R is said to be an **equivalence relation** on S if and only if for all x, y and z in S

- $(x, x) \in R$ (reflexive).
- If $(x, y) \in R$, then $(y, x) \in R$ (symmetric).
- If $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$ (transitive).

37. Equivalence class

If R is an equivalence relation on S, and $x \in S$, then the **equivalence class** of x is $\{y \in S \mid (x, y) \in R\}$.

38. Isomorphism class

A set of graphs isomorphic to each other is called an **isomorphism class** of graphs.

39. Automorphism

Given a graph X, a permutation α of $V(X)$ is an **automorphism** of X if

$$\{u, v\} \in E(X) \Leftrightarrow \{\alpha(u), \alpha(v)\} \in E(X), \text{ for all } u, v \in V(X).$$

40. Self-complementary

A **self-complementary** graph is a graph which is isomorphic to its complement.

41. Path and Cycle

- A **path** in a graph is a trail such that no internal vertex is repeated.
- A **cycle** is a closed path of length at least 1.

42. Walk, Trial and Internal vertex

- A **walk** in a graph G is an alternating sequence of vertices and edges,

$$W = v_0, e_1, v_1, e_1, \ldots, e_n, v_n,$$

  such that for $j = 1, \ldots, n$, the vertices v_{j-1} and v_j are the endpoints of the edge e_j.
- A **trail** in a graph is a walk such that no edge occurs more than once.
- A **u-v walk** is defined as a sequence of vertices starting at u and ending at v, where consecutive vertices in the sequence are adjacent vertices in the graph.
- A **u-v trail** is a u-v walk, where no edge is repeated (each edge is used at most once).
- A **u-v path** is a u-v walk, where no vertex is repeated (each vertex is used at most once).
- An **internal vertex** is a vertex of degree at least 2.

43. Length, Closed, Walk and Circuit

- The **length of a walk** is the number of edges (counting repetitions).
- A walk is **closed** if the initial vertex is also the final vertex; otherwise, it is **open**.
- A **circuit** is a closed trail with no repeated vertices except the initial and terminal ones.

44. Girth

The **girth** of a graph with a cycle is the length of its shortest cycle.

An acyclic graph has infinite girth.

45. Connected graph

    - A graph is **connected** if between every pair of vertices there is a walk. If not, it is **disconnected graph**.
    - **Connection relation** on $V(G)$ is $R = \{(u, v)|u, v$ is a walk$\}$.

46. Component and Trivial component

    A **component** of a graph G is a connected subgraph H such that no subgraph of G that properly contains H is connected. In other words, a component is a *maximal* connected subgraph.

    A component (or graph) is called **trivial** if there is consisting of one vertex and no edges.

47. Cut-edge and Cut-vertex

    A **cut-edge** (or **cut-vertex**) is an edge (or a vertex) whose removal increases the number of components.

48. Eulerian graph

    A graph is **Eulerian** if it has a closed walk that contains every edges exactly once. **Eulerian circuit** or **Euler tour** in an undirected graph is a cycle that uses each edge exactly once.

49. Odd and Even vertices

    - A graph vertex in a graph is said to be an **odd(even) vertex** if its vertex degree is odd(even).
    - An undirected graph is odd(even) if every vertex has odd(even) degree.

50. Graphical sequence

    We call a sequence of non-negative integers $d_1, \ldots, d_n$ **graphical** if there exists a graph G of order n the vertices of which have, in some order, degrees $d_1, \ldots, d_n$.