Combinatorial Designs Exercise 3 Due 2014, 5, 16

1. Prove that a λ-fold triple system of order v exists if and only if $\lambda(v-1)$ is even and $\lambda v(v-1)$ is a multiple of 6.

2. Prove that a GD[n, m: 3, 1] exists if and only if (a) $m \geq 3$, (b) $n(m-1)$ is even and (c) $n^2m(m-1)$ is a multiple of 6.

3. Prove that a $2 - (v, 4, 1)$ design exists if and only if $v \equiv 1$ or 4 (mod 12).

4. Prove that for each positive integer $n \neq 2$ or 6, there exists a pair of orthogonal Latin squares of order n.

5. Prove that if $v > k$ and a $2 - (v, k, \lambda)$ design exists which has b blocks, then $b \geq v$.