

1. Prove or disprove that a bipartite graph of size $2m$ contains an induced subgraph of size m .
2. Find $\max. \{|G| : G \text{ is a balanced bipartite graph of girth } 8.\}$ (*)
3. Find the crossing number of K_7 .
4. Prove that the Petersen graph can't be decomposed into three 1-factors, i.e. the Petersen graph is not 3-edge-colorable.
5. Prove or disprove that the complete graph of order 7 has a 2-cell embedding with exactly one region.
6. Find all k 's such that K_7 has a 2-cell embedding in S_k .