Algebra (I) Test 3 2013, 11, 29

1. Prove or disprove the following statements. (2 points each)

   (a) Let G be a group. Then $C = \{aba^{-1}b^{-1} \mid a, b \in G\}$ is a normal subgroup of G.

   (b) If K is a normal subgroup of H and H is a normal subgroup of G, then K is a normal subgroup of G.

   (c) If H is a normal subgroup of a finite group G, then the factor group of G over H, G/H, has $|G|/|H|$ elements.

   (d) Let H be a subgroup of G. Then G is an H-set under the action $*(h, g) = hgh^{-1}$ or $h*g = hgh^{-1}$.

   (e) Let G be a group which acts on the set X with group action $*$. Then for each $y \in X$, $G_y = \{g \in G: g*y = y\}$ is a subgroup of G.

2. Find a ring R in which the multiplication is not commutative. (Verify your answer.) (5 points)

3. (Bonus) Let G be a group which acts on the set X with group action $*$. Prove that for each $x \in X$, $|Gx| = |G|/|G_x|$ where $Gx = \{gx \mid g \in G\}$ and $G_x = \{g \in G: g*x = x\}$. (3 points)