

1. Prove or disprove the following statements. (2 points each)

(a)  $Z_3 \times Z_6$  is isomorphic to  $Z_{18}$ .

(b) If  $G$  is a group order 30 and  $H$  is a subgroup of  $G$  with 15 elements, then  $H$  is a normal subgroup of  $G$ .

(c) Any group of order 12 has a subgroup of order 6.

(d) The following two permutations on  $\{1, 2, 3, 4, 5, 6\}$  are exactly the same permutation:

$(123654), (12)(13)(16)(15)(14)$ .

(e) If  $\varphi$  is a homomorphism from  $G$  into  $G'$  (with identity  $e'$ ), then the kernel of the homomorphism,  $\text{Ker}(\varphi) = \{x \in G \mid \varphi(x) = e'\}$ , is a normal subgroup of  $G$ .

2. Give three different examples of “homomorphism” of groups.

(Verify your answers.) (5 points)

3. **(Bonus)** Prove that if  $G$  is a finite group of order  $m$  and  $H$  is a subgroup of  $G$  with  $n$  elements, then  $n$  is a divisor of  $m$ , i. e.

$n \mid m$ . (3 points)