Exercises(I) (詳細證明)

2013.10.01

1. Prove that every $2k$-regular graph can be decomposed into k 2-factors.

2. Let $n-1 \geq m_1 \geq m_2 \geq \cdots \geq m_t \geq 1$ such that $\Sigma_{i=1}^{t} m_i = \binom{n}{2}$. Prove that K_n can be decomposed into t paths $Z_1, Z_2, ..., Z_t$ such that $\|Z_i\| = m_i, i = 1, 2, ..., t$.

3. Prove that $C_6 | K_n$ if and only if $n \equiv 1$ or $9 \pmod{12}$.

4. Let $T_1, T_2, ..., T_{n-1}$ be either stars or paths such that $\|T_i\| = i$ for $i = 1, 2, ..., n-1$. Prove that $K_n = T_1 + T_2 + \cdots + T_{n-1}$.

5. Prove that every tree of size $\binom{n+1}{2}$ has an ASD.

6. Let T be a graph in Figure 1. Prove that K_{11} can be decomposed into 10 ascending subgraphs $G_1 \leq G_2 \leq \cdots \leq G_{10}$ such that $G_{10} \cong G$.