Cyclic Steiner Triple Systems

Rose Peltesohn [19] found the following difference triples. These can be used to form the base blocks (see Section 1.7) of a cylic $STS(v)$ for all $v \equiv 1$ or 3 (mod 6), $v \neq 9$. There does not exist a cyclic $STS(9)$.

v = 7 $\{1,2,3\}$
v = 13 $\{1,3,4\}$ and $\{2,5,6\}$
v = 15 $\{1,3,4\}$ and $\{2,6,7\}$
v = 19 $\{1,5,6\}$, $\{2,8,9\}$ and $\{3,4,7\}$
v = 27 $\{1,12,13\}$, $\{2,5,7\}$, $\{3,8,11\}$ and $\{4,6,10\}$
v = 45 $\{1,11,12\}$, $\{2,17,19\}$, $\{3,20,22\}$, $\{4,10,14\}$, $\{5,8,13\}$, $\{6,18,21\}$ and $\{7,9,16\}$
v = 63 $\{1,15,16\}$, $\{2,27,29\}$, $\{3,25,28\}$, $\{4,14,18\}$, $\{5,26,31\}$, $\{6,17,23\}$, $\{7,13,20\}$, $\{8,11,19\}$, $\{9,24,30\}$ and $\{10,12,22\}$.
v = 18s + 1, $s \geq 2$
$\{3r+1, 4s-r+1, 4s+2r+2\}$ for $0 \leq r \leq s-1$,
$\{3r+2, 8s-r, 8s+2r+2\}$ for $0 \leq r \leq s-1$,
$\{3r+3, 6s-2r-1, 6s+r+2\}$ for $0 \leq r \leq s-2$, and
$\{3s, 3s+1, 6s+1\}$.
v = 18s + 7, $s \geq 1$
$\{3r+1, 8s-r+3, 8s+2r+4\}$ for $0 \leq r \leq s-1$,
$\{3r+2, 6s-2r+1, 6s+r+3\}$ for $0 \leq r \leq s-1$,
$\{3r+3, 4s-r+1, 4s+2r+4\}$ for $0 \leq r \leq s-1$, and
$\{3s+1, 4s+2, 7s+3\}$.
v = 18s + 13, $s \geq 1$
$\{3r+2, 6s-2r+3, 6s+r+5\}$ for $0 \leq r \leq s-1$,
$\{3r+3, 8s-r+5, 8s+2r+8\}$ for $0 \leq r \leq s-1$,
$\{3r+1, 4s-r+3, 4s+2r+4\}$ for $0 \leq r \leq s$, and
$\{3s+2, 7s+5, 8s+6\}$.
v = 18s + 3, $s \geq 1$
$\{3r+1, 8s-r+1, 8s+2r+2\}$ for $0 \leq r \leq s-1$,
$\{3r+2, 4s-r, 4s+2r+2\}$ for $0 \leq r \leq s-1$, and
$\{3r+3, 6s-2r-1, 6s+r+2\}$ for $0 \leq r \leq s-1$.
v = 18s + 9, $s \geq 4$
$\{3r+1, 4s-r+3, 4s+2r+4\}$ for $0 \leq r \leq s$,

$\{3r+2, 8s-r+2, 8s+2r+4\}$ for $2 \leq r \leq s-2$,
$\{3r+3, 6s-2r+1, 6s+r+4\}$ for $1 \leq r \leq s-2$,
$\{2, 8s+3, 8s+5\}$, $\{3, 8s+1, 8s+4\}$, $\{5, 8s+2, 8s+7\}$,
$\{3s-1, 3s+2, 6s+1\}$, and $\{3s, 7s+3, 8s+6\}$.
v = 18s + 15 $s \geq 1$
$\{3r+1, 4s-r+3, 4s+2r+4\}$ for $0 \leq r \leq s$,
$\{3r+2, 8s-r+6, 8s+2r+8\}$ for $0 \leq r \leq s$, and
$\{3r+3, 6s-2r+3, 6s+r+6\}$ for $0 \leq r \leq s-1$.