Graph Theory(II), Homework(I)

Due 2008, 3, 31

1. Prove that G is planar if and only if neither K_5 nor $K_{3,3}$ is a minor of G.

2. Prove that every 3-connected graph with at least 6 vertices that contains a subdivision of K_5 also contains a subdivision of $K_{3,3}$.

3. Estimate the thickness of $K_{m(n)}$ by giving an upper bound and a lower bound respectively.

4. Estimate the crossing number of $K_{3,3,3}$.

5. Find a 2-cell embedding of K_6 on the surface S_t for all possible t.

6. Let the genus of G be n. Then, prove that any embedding of G on S_n is a 2-cell embedding.

7. Draw an embedding (2-cell) of K_8 on S_2.

8. Prove or disprove that K_7 has a 2-cell embedding on N_2.

9. Prove that $\chi(S_n) = \left\lfloor \frac{7+\sqrt{1+48n}}{2} \right\rfloor$ for each positive integer n.

10*. Find the genus of n-cube, i.e., $\gamma(Q_n)$.