GraphTheory(II), Exercise(II)

Due May 30, 2007.

1. Prove that $ex(n;K_{r+1)}) = \|T_r(n)\|$ and $T_r(n)$ is the unique extremal graph.(Give at least two different proofs of the first part.)

2. Prove that $ex(n;K_{s,t}) \leq \dfrac{1}{2}(s-1)^{\frac{1}{t}}n^{2-\frac{1}{t}} + \dfrac{1}{2}(t-1)n$, $t \leq s$.(Give at least two different proofs.).

3. For infinitely many positive integers n, find a graph G of order n such that $K_{2,2} \not\leq G$ and $\|G\| \approx \dfrac{1}{2}n^{\frac{3}{2}}$.

4. Find a C_4-saturated graph of order n with minimum number of edges for each positive integer n.

5. Find a $K_{2,3}$-saturated graph of order n with minimum number of edges.

6. Find $z(n,n;2,2)$ for as many n as possible.

7. For each positive integer t, $20 \leq t \leq 36$, find a subgraph G of $K_{9,12}$ which is C_4-saturated and $\|G\| = t$.

8. For every graph H with at least one edge, prove that $\lim_{n\to\infty} \dfrac{ex(n;H)}{\binom{n}{2}} = \dfrac{\chi(H)-2}{\chi(H)-1}$.

9. Given n points in the plane, prove that the distance is exactly 1 for at most $\frac{1}{\sqrt{2}}n^{\frac{3}{2}} + \frac{n}{4}$ pairs.

10. Construct a graph G of order 11 such that $K_{3,3} \not\leq G$ and G has at least $\lfloor \frac{1}{2}(11^{\frac{5}{3}}) \rfloor$ edges.