GraphTheory(II), Exercise(I)

Due Apr. 18, 2007.

1. Prove that a graph is planar if and only if it does not contain a Kuratowski subgraph.
2. Use Kuratowski's Theorem to prove that G is planar if and only if neither K_5 nor $K_{3,3}$ is a minor of G.
3. Prove that every 3-connected graph with at least 6 vertices that contains a subdivision of K_5 also contains a subdivision of $K_{3,3}$.
4. Find the thickness of K_n.
5. Find the thickness of $K_{n,n}$.
6. Find the crossing number of $K_{3,3,3}$.
7. Find the crossing number of $K_{m,n}$.
8. Let G be a (p, q)-graph which has a 2-cell embedding on the surface S_γ with r regions. Prove that $p-q+r = 2-2\gamma$.
9. Find a 2-cell embedding of K_6 on the surface S_t for each $t \in \{2, 3, 4, 5\}$.
10. Prove that if G is embeddable on S_γ, then G has at most $3(p - 2 + 3\gamma)$ edges where $|G| = p$.