GraphTheory(I), Homework(III)

Due Jan. 4, 12:00.

1. For positive integers $p \geq 10, k = 2, 3, 4$, prove that C_p^k can be decomposed into k Hamiltonian cycles.

2. Let D be a directed graph such that for each $v \in V(D)$, $deg_D^+(v) = deg_D^-(v) = t \geq 1$. Prove that D can be decomposed into t directed 1-factors (with indegree 1 and outdegree 1).

3. Prove that G is a perfect graph if and only if $\overline{G}$ is a perfect graph.

4. Prove that $\chi'(G) \leq 3\Delta(G)/2$.

5. Prove that if $\|G\| = \binom{n+1}{2}$ and $\Delta(G) \leq \lfloor n/2 \rfloor$, then G can be decomposed into n subgraphs $G_1, G_2, \cdots, G_n$ such that G_i is induced by a matching with i edges for all i.

6. Let D be an n-regular digraph of order $2n+1$, $n \geq 1$. Prove or disprove that D has a directed Hamiltonian cycle. (D is n-regular if $\forall v \in V(D), d^+(v) = d^-(v) = n$.)

7. Let $n = k(2l+1)$. Construct a non-Hamiltonian complete k-partite graph with n vertices and minimum degree $\frac{n}{2}\frac{k-1}{k}\frac{2l}{2l+1}$. Ex. 7.2.21

8. Dirac proved that every 2-connected simple graph G has a cycle of length at least $\min\{n(G), 2\delta(G)\}$. Use this to prove that every $2k$-regular graph with $4k+1$ vertices is Hamiltonian. Ex. 7.2.40

9. (a) Use Vizing's Theorem to prove that $\chi'(G \square K_2) = \Delta(G \square K_2)$.
   (b) Let G_1, G_2 be edge-disjoint graphs with vertex set V, and let H_1, H_2 be edge-disjoint graphs with vertex set W. Prove that $(G_1 \cup G_2) \square (H_1 \cup H_2) = (G_1 \square H_2) \cup (G_2 \square H_1)$.
   (c) Prove that $\chi'(G \square H) = \Delta(G \square H)$ if both G and H have 1-factors. Ex. 7.1.25

10. Prove that if G is a multigraph with multiplicity μ, then $\chi'(G) \leq \Delta(G) + \mu$.