GraphTheory(I), Homework(I)

Due Oct. 18, 12:00.

1. Let a rectangle T be tiled with rectangles $T_1, T_2, \cdots, T_l$. Prove that if each T_i has an integral side, then T also has an integral side. (Use as many distinct ways to prove it as you can.)
2. Prove that a disconnected graph is reconstructible.
3. Use adjacency matrix of G to find the number of 5-cycles in G.
4. Prove that every bridgeless cubic graph has a 1-factor.
5. Prove that for each $g \geq 3$, a $(3, g)$-cage has no bridge.
6. Prove or disprove that if $k \geq 2$ and $k \left| \binom{n}{2} \right.$, then the edge set of K_n can be partitioned into k sets $E_1, E_2, \cdots, E_k$ such that they induce k isomorphic subgraphs of K_n.
7. Hikers A and B begin at $(a, 0)$ and $(b, 0)$, respectively. Prove that A and B can meet by travelling on the mountain range in such a way that at all times their heights are the same. (Hint: Define a graph to model the movements.) Ex. 1.3.13 (D. B. West)
8. For $n \geq 3$, determine the minimum number of edges in a connected n-vertex graph in which every edge belongs to a triangle. Ex. 1.3.50 (D. B. West)
9. Let $d_1 \leq d_2 \leq \cdots \leq d_n$ be the degrees of a simple graph G. Prove that G is connected if $d_j \geq j$ when $j \leq n - 1 - d_n$. (Hint: consider a component that omits some vertex of maximum degree.) Ex. 1.3.64 (D. B. West)
10. Let S be an n-element set, and let $\{A_1, \cdots, A_n\}$ be n distinct subsets of S. Prove that S has an element x such that $A_i \cup x$ are distinct for all i. Ex. 2.1.76 (D. B. West)